Finally, either the value $\hat{\sigma}^2$, which limits the stress, or the function $\sigma_y^{0^2}(T)$ should be substituted for $\hat{\delta}^2$ in (9).

This statement of the problem is, however, less effective than the preceding one. Moreover, its use presupposes knowledge of all the mechanical characteristics of the material in question, which is not always the case [8]. In connection with this, it may prove useful to state inverse problems in a manner similar to that used for determining the current in section 3 and solve them by means of a certain regularizing algorithm.

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THERMAL AND THERMODEFORMATIONAL PROCESSES IN THE FORCED HEATING OF STEEL

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The dynamics of metal heating in a furnace of pacing-beam type is investigated experimentally and theoretically.

The heating of metal in a furnace is investigated experimentally, with the aim of subsequent parametric identification of the mathematical model, for the example of steel-15 blooms of cross section 250 × 300 mm.

In the course of the industrial experiment, the temperature values of control points of the cross section of the experimental ingot (corner, surface, center) is determined from the instant of insertion to the removal of the metal, as well as the degree of oxidation of surface layers of the steel. The temperature is measured using KhA thermocouples with an electrode diameter of 1.2 mm. The productivity of the heating furnace in the experiment is 46.7 ton/h.

Note that the presence of a positive static pressure in the working space of the furnace eliminates the possibility of cold-air inflow. Ignition of the fuel with a consumption coefficient of 1.0-1.1 in these conditions creates an atmosphere with weak oxidative properties. This is confirmed by the experimental results: the degree of oxidation of the metal is no more than 1%.

Analysis of the components of the thermal balance allows the efficiency of the furnace and the specific consumption of the conventional fuel to be determined: 61.5% and 35.2 kg of fuel/ton of steel, respectively. The total heat losses through the load with cooling water and with incomplete chemical combustion are no more than 10%, which indicates high efficiency of operation of the furnace.

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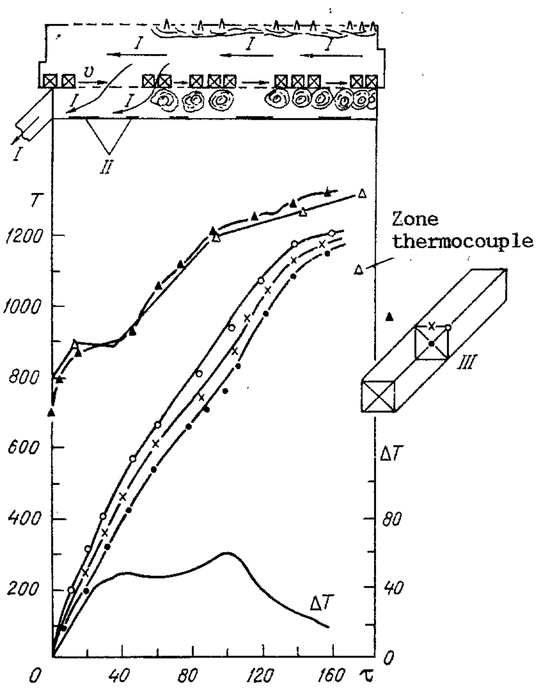

Fig. 1. Parametric identification of the mathematical model of bloom heating in a furnace of pacing-beam type from the results of an industrial experiment: I) dusty gas; II) idle intervals; III) position of thermocouple in ingot.

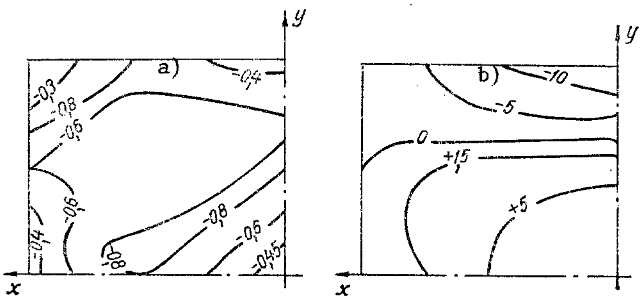

Fig. 2. Distribution of stress-state coefficient (a) and thermostress (σ_y, kg/mm²) (b) in cross section of bloom (high-speed heating).

The mathematical model of the heating of an ingot of rectangular cross section includes the two-dimensional heat-conduction equation

$$\rho(T)\,c(T)\,\frac{\partial T}{\partial \tau}=\frac{\partial}{\partial x}\left(\lambda(T)\,\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda(T)\,\frac{\partial T}{\partial y}\right), \tag{1}$$

$$-R_1\leqslant x\leqslant R_1,\quad -R_2\leqslant y\leqslant R_2,\quad 0\leqslant\tau<\infty,$$

and the boundary and initial conditions

$$\lambda(T)\,\frac{\partial T(R_1,\,y,\,\tau)}{\partial x}=\frac{\sigma}{10^8}\,[T_{\mathrm{fu}}^4-T^4(R_1,\,y,\,\tau)]+\alpha\,[T_{\mathrm{fu}}-T(R_1,\,y,\,\tau)], \tag{2}$$

$$\lambda(T)\,\frac{\partial T(x,\,R_2,\,\tau)}{\partial x}=\frac{\sigma}{10^8}\,[T_{\mathrm{fu}}^4-T^4(x,\,R_2,\,\tau)]+\alpha\,[T_{\mathrm{fu}}-T(x,\,R_2,\,\tau)], \tag{3}$$

$$\frac{\partial T(0,\,y,\,\tau)}{\partial x}=0,\quad \frac{\partial T(x,\,0,\,\tau)}{\partial y}=0,\quad T(x,\,y,\,0)=f(x,\,y), \tag{4}$$

where T_{fu} is the furnace temperature measured in the course of the experiments by regular zone thermocouple and thermosensors introduced in the working space of the furnace at the surface of the heated ingot; σ, α are the effective coefficients of radiation and convective heat transfer at the ingot surface. The parameter α, which has little influence on the calculation results, is taken to be 30 W/m²·K, in accordance with literature data. The parameter σ is very undetermined, and is found by theoretical—experimental identification. As is evident from Fig. 1, identification allows a thermal model of the process of high accuracy to be obtained. The identified values of σ vary from zone to zone within limits of 1.7-1.9 W/m²·K.

In developing the mathematical model for calculating the thermoelastoplastic stress and deformation, it is assumed that the temperature distribution in the ingot does not depend on the axial coordinate z. Then, in accordance with the hypothesis of plane cross sections

$$\sigma_{xz} = \sigma_{yz} = \varepsilon_{xz} = \varepsilon_{yz}.$$

In this case, the system of equations of the model written within the framework of flow theory in increments takes the form

$$\frac{\partial \Delta\sigma_x}{\partial x} + \frac{\partial \Delta\sigma_{xy}}{\partial y} = 0, \quad \frac{\partial \Delta\sigma_y}{\partial y} + \frac{\partial \Delta\sigma_{xy}}{\partial x} = 0, \tag{5}$$

$$\begin{aligned}
\Delta\varepsilon_x &= c_{11}\Delta\sigma_x + c_{12}\Delta\sigma_y + c_{13}\Delta\sigma_z + c_{14}\Delta\sigma_{xy} + \varphi_x\Delta T,\\
\Delta\varepsilon_y &= c_{21}\Delta\sigma_x + c_{22}\Delta\sigma_y + c_{23}\Delta\sigma_z + c_{24}\Delta\sigma_{xy} + \varphi_y\Delta T,\\
\Delta\varepsilon_z &= c_{31}\Delta\sigma_x + c_{32}\Delta\sigma_y + c_{33}\Delta\sigma_z + c_{34}\Delta\sigma_{xy} + \varphi_z\Delta T,\\
\Delta\varepsilon_{xy} &= c_{41}\Delta\sigma_x + c_{42}\Delta\sigma_y + c_{43}\Delta\sigma_z + c_{44}\Delta\sigma_{xy} + \varphi_{xy}\Delta T,
\end{aligned} \tag{6}$$

$$\frac{\partial^2 \Delta\varepsilon_{xy}}{\partial x \partial y} = \frac{\partial^2 \Delta\varepsilon_x}{\partial y^2} + \frac{\partial^2 \Delta\varepsilon_y}{\partial x^2}, \tag{7}$$

$$\frac{\partial^2 \Delta\varepsilon_z}{\partial x^2} = 0, \quad \frac{\partial^2 \Delta\varepsilon_z}{\partial y^2} = 0, \quad \frac{\partial^2 \Delta\varepsilon_z}{\partial x \partial y} = 0, \quad \Delta u|_{x=R_1} = 0, \quad \Delta v|_{y=R_2} = 0, \tag{8}$$

where Δu, Δv are the increments in the displacements along the x and y axes; $\Delta\sigma_x$, $\Delta\sigma_y$ are the components of the stress increment; $\Delta\varepsilon_x$, $\Delta\varepsilon_y$ are the deformations; ΔT is the temperature increment at the given point of the body after the loading step; the coefficients c_{ij} depend on the accumulated deformation and stress; the dependence of the elastic and plastic properties of the material on the temperature is calculated by the method of [1]. In accordance with the principle of constructing a model of a generalized plane deformed state [1], equilibrium conditions should be added to the system in Eqs. (5)-(8)

$$\int_F \Delta\sigma_z dF = 0, \quad \int_F \Delta\sigma_z x dF = 0, \quad \int_F \Delta\sigma_z y dF = 0, \tag{9}$$

where the integrals are taken over the cross-sectional area of the ingot.

The solution of the complete system in Eqs. (5)-(9) in each loading step is based on the procedure of [1], using the finite-element method.

With the aim of estimating the upper bound on the thermostress in the ingot, the thermal and thermodeformational processes are calculated in forced-heating conditions. The rate of heating is specified as the maximum permissible value in all the zones; the initial ingot temperature is assumed to be 20°C. The degree of risk of crack appearance is estimated using the criterion of [2], according to which cracks develop when the stress intensity σ_i at the given point of the body exceeds the experimental critical value σ_T, which depends on the temperature. At least one component of the stress must be tensile here. Therefore, to estimate the stress state, as well as the stress field, the stress-state coefficients $k_\sigma = (\sigma_i - \sigma_T(T))/\sigma_T(T)$ are calculated. It is clear that the risk of crack appearance increases with increase in k_σ, especially if $k_\sigma > 0$.

The theoretical variation in the stress σ_y and k_σ at the time corresponding to the appearance of maximum k_σ is shown in Fig. 2. The greatest value of k_σ is seen at the center of the ingot and at the surface in the middle of the faces.

The results obtained indicate that, even in forced-heating conditions, the tensile stress does not reach critical values leading to crack appearance. Note that the problem is solved in the elastoplastic approximation and, since viscous effects begin to exert an influence in conditions of high-temperature heating, the real values of the stress and hence k_σ

must be lower. Thus, the stress appearing in high-speed heating of the blooms cannot lead to loss of continuity of the metal and hence there is a possibility of increasing the design productivity of the apparatus.

NOTATION

T, temperature; ρ, c, λ, thermophysical coefficients; α, σ, external heat-transfer coefficients; τ, time; R, characteristic dimension; σ, thermal stress in ingot cross section; ε, deformation; Δu, Δv, increments in displacements over the x, y axes; $k_\sigma = (\sigma_i - \sigma_T(T))/\sigma_T(T)$, stress-state coefficient.

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MODELING MACROSEGREGATION IN AN INGOT, TAKING ACCOUNT OF ALLOY SHRINKAGE

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A mathematical model is proposed for the description of zonal segregation in an ingot of quiescent steel on casting in a mold, taking account of alloy shrinkage.

Despite the development of continuous-casting technology, most the steel forged in the USSR is cast in a mold. Both technologies have a common deficiency: the larger the ingot, the more impurity segregation occurs on solidification. The mechanism of zonal-segregation formation was described in [1-4], and experimental data have been obtained on the zonal segregation of various chemical elements on casting. However, in our view, no sufficiently complete method of calculating solidification [5, 3], taking account of melt flows and maintenance of the liquid core of the ingot on alloy shrinkage [1, 2], exists as yet. A mathematical model is created in the present work for the complex investigation of these processes. It is developed for the calculation of the thermal and concentrational fields and consists of a system of differential equations with the corresponding boundary conditions.

The liquid-phase motion in the solidifying ingot is a superposition of the flux due to large-scale processes (such as stratification, mixing, etc.) and microfluxes arising on filling of the shrinkage volumes. First of all, the shrinkage component of the liquid-phase velocity is isolated, by writing the continuity equation. The metal density is a superposition of the densities of the solid ρ_s and liquid ρ_ℓ phases, which are not equal but constants: $\rho = \rho_s\varphi + \rho_\ell(1-\varphi)$.

Using the mixing rates of the solid and liquid phases v_{sh} and $v_{\ell h}$, which satisfy the incompressibility equation, and also the shrinkage component of the liquid-phase velocity $v_{\ell y}$, the continuity equation is written in the form

$$\rho_s\nabla(v_{sh}\varphi) + \rho_l\nabla(v_{lh}(1-\varphi)) = 0, \qquad (1)$$

and, taking into account that $d\rho/d\tau = 0$, it follows that

$$\frac{\partial(\rho_s\varphi)}{\partial\tau} + \rho_s\nabla(v_{sh}\varphi) + \frac{\partial(\rho_l(1-\varphi))}{\partial\tau} + \rho_l\nabla[v_{lh}(1-\varphi) + v_{ly}(1-\varphi)] = 0. \qquad (2)$$

Hence, substituting Eq. (1) in Eq. (2), an equation determining the shrinkage rate $v_{\ell y}$ may be obtained

$$(\rho_s - \rho_l)\frac{\partial\varphi}{\partial\tau} + \rho_l\nabla[v_{ly}(1-\varphi)] = 0. \qquad (3)$$

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